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XXVIII. *On the Velocity of Sound in Air, Gases, and Vapours for Pure Notes of different Pitch.* By J. WEBSTER LOW, *Ph.D., B.A.**

§ 1. *Introduction.*

BY the publication of Regnault's† great work and the immediate corroboration of his results by Le Roux‡, the general confidence in the previously accepted value of the velocity of sound was severely shaken. Since then several experimenters have sought, by measuring the wave-lengths of notes of different pitch, to arrive indirectly at the velocity of sound. With this object Kundt§ and Kayser|| have utilized the former's dust-figures, Schneebeli ¶ and Seebeck** Quincke's interference-tubes††; and all have agreed in finding a greater velocity for the higher notes than for the lower ones, a result the reverse of that found by Regnault and König‡‡.

From theoretical considerations, Helmholtz§§ and Kirchhoff|||| have shown that friction and the conduction of heat

* Communicated by the Author.

† *Compt. Rend.* lxvi. pp. 209–220; *Mém. de l'Inst.* xxv. (1867).

‡ *Ann. de Chem.* 4 série, xii. pp. 345–418.

§ Pogg. *Ann.* cxxvii. p. 497 (1866); and cxxxv. pp. 337–372 and 527–561 (1868).

|| Wied. *Ann.* ii. pp. 218–241 (1877); and vi. p. 465 (1879).

¶ Pogg. *Ann.* cxxxvi. p. 296 (1869).

** Pogg. *Ann.* cxxxix. p. 104 (1870).

†† Quincke, Pogg. *Ann.* cxxviii. p. 177 (1866).

‡‡ König, *Mém. de l'Inst.* xxxvii. p. 435.

§§ *Verhandlungen des natur.-histor. medicin. Vereins zu Heidelberg vom Jahre* 1863, iii. p. 16.

|||| Pogg. *Ann.* cxxxiv. p. 177 (1868).

must greatly affect the wave-length and the velocity of sound in narrow tubes. Their theory agrees only imperfectly with the results of Kundt, who employed mixed notes; with those of Schneebeli, Seebeck, and Kayser, however, all of whom used pure musical tones, the accord is somewhat better. The methods of the last named inquirers, though correct in principle, are, however, in detail liable to various objections; I have therefore subjected the whole question of the indirect determination of the velocity of sound to a fresh investigation.

The questions I set myself for answer were:—

1. How does the velocity of sound vary in air and gases for pure notes of different pitch in tubes of different diameter?

2. How can the true velocity of sound in unlimited space be determined from that found in tubes?

§ 2. *Method of the Inquiry.*

At the suggestion of Prof. Quincke I measured the wave-lengths for tuning-forks of known vibration-frequency by means of his interference-tubes. I observed, not one minimum of vibration-intensity, as Seebeck had done, but successive maxima, by shortening the tubes by one, two, or more half wave-lengths. My apparatus (fig. 1) consisted of a wide glass tube O U, closed at the bottom with a cork and a stop-cock H. From H a long piece of guttapercha tubing led to a water-bottle F; a second piece of narrower tubing connected the side-tube A, distant about 5 centimetres from the upper end of the main tube, with the ear of the observer at C, ending in a glass pipe coated with sealing-wax, so as to fit exactly into the outer passage of the ear. By raising and lowering the bottle F a swimmer B could be brought to any desired point of the interference-tube, and the exact position of its upper smooth surface could be read off on a millimetre-scale fixed behind the tube. The swimmer consisted of a cork 4 centim. long, loaded at the lower end with lead and coated with stiff paper and paraffin. The cork had almost the same diameter as the tube.

The theory of vibrating air-columns, as developed by Kirchhoff, postulates a regular motion of the air-particles parallel to the axis of the tube. In the course of my experiments, however, whether the bare prong or the resonance-box of the tuning-fork was held over the opening of the tube, or the fork-handle pressed firmly against any point of its sides, or the fork, with box attached, removed to any part of the room, I failed to observe any change in the positions of the maxima. These positions could, however, be most easily and distinctly found by using the flat side of the bare prong. Some of the earliest readings were taken with the resonance-box; but I soon laid it aside, as the strong resonance of the box obscured too much the maximum of resonance of the tube.

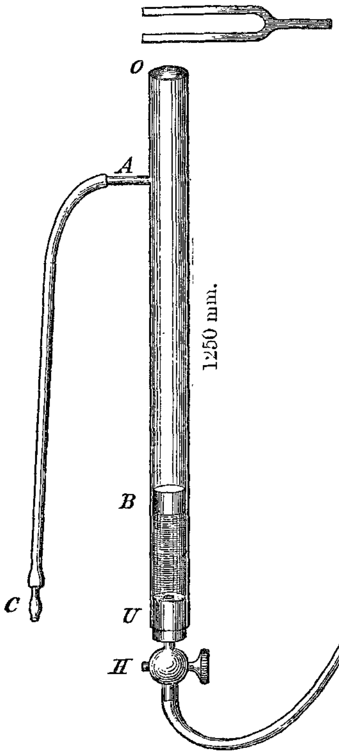

Fig. 1.

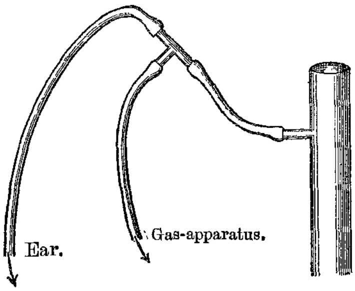

Fig. 2.

In a tube like that represented in fig. 1 the air-column can resonate in two different ways. In the one case a loop forms at the side-piece A, and a minimum of intensity is heard through the tubing A C. The distance of the loop from the reflecting surface of the swimmer is then an odd multiple of a quarter wave-length. Raise the swimmer through a distance equal to the length A O, increased by the amount of Rayleigh's correction* for open pipes, then a loop forms at O, a greater change of density takes place at A, and a maximum of intensity is heard through A C. The distance of the

* $+0{\cdot}82 \times$ radius of the pipe. *Vide* Lord Rayleigh's 'Sound,' vol. ii. § 307, and Appendix A.

swimmer now from the end O, increased by Rayleigh's correction, is an odd multiple of a quarter wave-length. The difference between two successive maxima or minima is thus half a wave-length. After careful trial I found that the maxima, at all events for my ear, could be fixed with much greater precision than the minima.

From the observed half wave-length, v_0, the velocity of sound in dry air at 0° temperature and 760 millim. pressure was calculated by means of the formula

$$v_0 = 2n\frac{\lambda}{2}\left(1 - \frac{\alpha t}{2} - \frac{3}{16}\frac{\mathrm{S}}{\mathrm{B}}\right);$$

where

n = the vibration-frequency of the fork,
λ = the observed wave-length,
α = the coefficient of expansion of air,
t = the temperature of the air in degrees Cent.,
S = the vapour-tension of water,
and B = the barometric height.

Any influence of the intensity or *vis viva* of vibration upon the velocity of sound, which Regnault* thought he had discovered, but which Rink† has with justice disputed, I could not observe. Kundt* also and Kayser* have found the velocity of sound invariable for different degrees of vibration-energy of the sounding body.

In making my observations I proceeded in the following manner:—With the water-bottle in the hand I raised and lowered the swimmer a few times until I had with tolerable certainty ascertained the positions of the maxima. More than one reading of the same maximum at the same time I never took; I rather returned to it four or five times in order to get my observations as independent of one another as possible.

The temperature was observed above and below at the beginning and the end of each experiment.

The tuning-forks $c_{\prime}$, $e_{\prime}$, $g_{\prime}$, $c_{\prime\prime}$, $c_{\prime\prime\prime}$ were all made by König, of Paris. The vibration-frequencies of the first four were found to be in the ratio of 4 : 5 : 6 : 8, but $c_{\prime\prime\prime}$ made 1023·25 vibrations when $c_{\prime\prime}$ made 512.

I used three different tubes, which were of the same length, 1250 millim., and of which the diameters were 28 millim., 17·1 millim., and 9·35 millim.

In order to convey a clear idea of the degree of precision of which the method is capable, I shall quote the details of a

* Regnault, Kundt, Kayser (see footnotes, *ante*, p. 249).
† Pogg. *Ann.* cxlix. p. 546 (1873).

single experiment taken at random from my journal. This was one of a series made at the beginning of the inquiry, when I had joined two tubes together in order to follow the maxima as far down into the tube as possible. As was to be expected on account of the decreasing intensity, the readings became more difficult the farther they were removed from the source of sound. After convincing myself in this way that the wave-length remained constant, I determined only the uppermost maximum and the lowermost that could be distinctly observed, divided their difference by the number of half wave-lengths contained in the interval passed through, and arrived in this way at the mean half wave-length.

Fork c_{ii}, n	$=512$.
Tube II., diameter .	$=17{\cdot}1$ millim.
Mean temperature .	$=12^{\circ}{\cdot}3$ C.
Vapour-tension . .	$=10{\cdot}5$ millim.
Barometric height .	$=754$ millim.

Readings for the Maxima.

Temp. C.	1.	2.	3.	4.	5.	6.	7.
	millim.	millim.	millim.	millim.	millim.	millim.	millim.
12	157	483	811	1136	1464	1792	2122
12·3	157	484	812	1138	1466	1793	2119
12·3	156·5	484	811	1139	1466	1792	2121
12·7	157·5	485	812	1140	1467	1794	2121
Mean 12·3	157*	484	811·5	1137·7	1466·3	1792·7	2120·7

Half Wave-lengths.	Velocity.
327 millim.	326·3 metre.
327·5 „	326·9 „
326·2 „	325·6 „
328·6 „	327·8 „
326·4 „	325·8 „
328 „	327·4 „
Mean	326·6 „

* The first quarter wave-length in the above experiment, increased by Rayleigh's correction (157+0·82 radius), gives a velocity of 327·4 metre. I very often calculated the velocity in this way, and always found nearly the same value as from the other readings. Such values were, however, never included in the mean.

From the above figures it is plain that the method, even for a tyro, makes considerable pretensions to exactness; after many months of practice, however, the limits of error became still closer.

Table showing the observed Mean Velocities in Air.

	Internal diameter of the tube.	$c_{,}$. n=256.	$e_{,}$. n=320.	$g_{,}$. n=384.	$c_{,,}$. n=512.	$c_{,,,}$. n=1023·25.
	millim.	metre.	metre.	metre.	metre.	metre.
I.	28	327·29	327·50	327·69	328·33	328·68
II.	17·1	325·24	325·54	326·03	326·70	327·80
III.	9·35	320·60	321·19	321·88	323·60	325·29

§ 3. Kirchhoff's *Formula discussed.*

In the light of these results let us test Kirchhoff's* theoretical formula for the velocity of sound in tubes:—

$$v=a\left(1-\frac{\gamma}{2r\sqrt{\pi n}}\right),$$

where

v = the observed velocity in tubes,
a = the velocity in unlimited space,
r = the radius of the tube,
n = the vibration-frequency of the tuning-fork,
and γ = a constant for friction and conduction of heat.

For the same tube the product $(a-v)\sqrt{n}$ must be constant, as also $(a-v)2r$ for the same tone.

Then from two results with the same fork and different tubes we get

$$a=\frac{v_1r_1-v_2r_2}{r_1-r_2},$$

where v_1 and r_1 denote the velocity and the radius of the wider tube, v_2 and r_2 the same quantities for the narrower one. Thus, by combining in pairs the results contained in the vertical columns of the above table, we should always get the true velocity of sound in the open air. My results calculated in this way are as follows:—

* Pogg. *Ann* cxxxiv. p. 177 (1868); or Kirchhoff's *Ges. Abh.* p. 543.

Tuning-fork.	Tubes I. and II.	Tubes I. and III.	Tubes II. and III.	Mean.
	metre.	metre.	metre.	metre.
$c_{,}$	330·3	330·6	331·1	330·67
$e_{,}$	330·3	330·6	330·9	330·60
$g_{,}$	330·1	330·5	330·9	330·50
$c_{,,}$	330·7	330·7	330·6	330·67
$c_{,,,}$	330·3	330·5	330·6	330·47
			Mean	330·582

If this be the true velocity of sound in the open, then the values of $(a-v)2r$ taken vertically in the first of the following tables, and those of $(a-v)\sqrt{n}$ taken horizontally in the second, should, as required by Kirchhoff's formula, be constant.

Table I.

Values of $(a-v)2r$.

Internal diameter of the tube.	$c_{,}$. $n=256$.	$e_{,}$. $n=320$.	$g_{,}$. $n=384$.	$c_{,,}$. $n=512$.	$c_{,,,}$. $n=1023{\cdot}25$.
millim. 28	0·09843	0·08626	0·08093	0·06301	0·05321
17·1	9825	8618	7780	6634	4753
9·35	9867	8778	8132	6527	4945

Table II.

Values of $(a-v)\sqrt{n}$.

Internal diameter of the tube.	$c_{,}$. $n=256$.	$e_{,}$. $n=320$.	$g_{,}$. $n=384$.	$c_{,,}$. $n=512$.	$c_{,,,}$. $n=1023{\cdot}25$.
millim. 28	52·64	55·09	56·62	50·91	60·77
17·1	85·43	90·14	89·15	87·78	88·90
9·35	159·7	168·0	170·4	158·0	169·2

Considering that any error in the observed value of v becomes greatly magnified in the above numbers, the small deviations from a constant mean are almost negligible. The generally excellent agreement between theory and experiment, when $a=330{\cdot}58$ metre, speaks for the correctness of this number as the true value of the velocity of sound.

If with this value we calculate the constant γ for the different tubes, we find the following results :—

Tube.	$c_{,}$.	$e_{,}$.	$g_{,}$.	$c_{,,}$.	$c_{,,,}$.
I.	0·007902	0·008265	0·008500	0·007642	0·009122
II.	7830	8262	8264	8046	8149
III.	8002	8418	8543	7916	8480
Mean=0·007989.					

This experimental mean 0·007989 tallies very closely with 0·00742, the theoretical value, calculated by means of Kirchhoff's formula * from O. E. Meyer's † constant of friction of air and Maxwell's theory of the conduction of heat.

We may also calculate k, the ratio of the specific heats, by the formula

$$a=\sqrt{\frac{\mathrm{B}\,.\,\mathrm{Q}\,.\,g\,.\,k}{\Delta_0}},$$

where $\mathrm{B}=0{\cdot}760$ metre, $\mathrm{Q}=13{\cdot}596$, $g=9{\cdot}81$ metre, and $\Delta_0=\frac{1}{773}$.

By substituting these values and putting $a=330{\cdot}582$ metre we find

$$k=1{\cdot}3947,$$

while all previous values lie between 1·419 and 1·3845.

*
$$\gamma=\sqrt{\mu}+\left(\frac{a}{b}-\frac{b}{a}\right)\sqrt{\nu},$$

where

a = the true value of the velocity of sound in air,
b = Newton's " " "
μ = a constant for conduction of heat,
ν = a constant for friction.

Vide Kirchhoff's *Ges. Abh.* p. 543.

† Pogg. *Ann.* xxxii. p. 642.

My results for air may be summed up as follows :—

1. The velocity of sound in narrow glass tubes is smaller than in the open air; it increases with the diameter of the tube and the pitch of the note.

2. The loss which the velocity of sound suffers in narrow glass tubes is inversely proportional to the diameter and the square root of the vibration-frequency. In other words, the formula

$$a-v=\frac{\gamma}{2r\sqrt{\pi n}}$$

is correct if a, the true velocity, $=330{\cdot}582$ metre*, and γ, the constant for friction and conduction of heat, $=0{\cdot}007989$.

3. The ratio of the specific heats for air is $1{\cdot}3947$*.

§ 4. *Carbonic Acid.*

The alteration of the apparatus necessary for the application of this method to gases depends upon the density of the gas.

For carbonic acid everything remained the same as for air, with the exception of a small change at the side-piece A.

As the main tube had to remain open, the chief difficulty lay in keeping the gas pure. The air could either, by diffusion, penetrate into the tube, or, by the lowering of the water-column, be drawn into it. The difficulty was overcome in the following manner.

The three arms of a T (fig. 2) were fitted with pieces of gutta-percha tubing, of which the one led to the side tube A, the other to the gas-apparatus, and the third to the ear of the observer. The T-piece was fixed so high and so inclined to one side, that the heavy gas flowed in a natural manner into the main tube.

The gas was generated in a Kipp's apparatus from oyster-shells and hydrochloric acid, and conducted through a system of wash-bottles and pearl-tubes saturated with a solution of sodium carbonate. The method of filling was as follows :— The swimmer was raised to A and the stopcock H closed. After the gas had flowed a short time through the ear-tubing, it was firmly clamped. The gas could now only escape through the side piece A, and thus the air still remaining in the upper portion of the main tube was rapidly expelled. The open end at O was then made air-tight and the stopcock H opened. The energy of the generation of the gas drove the water out of the tube back into the bottle, but always against a small counter-pressure, as I always took care that the surface

* Corrected on page 264.

of the water in the bottle stood higher than the surface of that in the tube. In this way it was impossible for the air to penetrate into the tube, even though the apparatus had not been air-tight. Everything was, however, always perfectly air-tight. When the tube was full, the stopper at O and the clamp on the ear-tubing were removed, the tuning-fork bowed, and the positions of the maxima ascertained exactly as in the case of air. While the observations were being made, the gas apparatus remained in action, so that a slow steady stream of pure carbonic acid poured through the upper portion of the tube and overflowed its edges.

The observed half wave-length was corrected to 0° C. and 760 millim. in dry gas by means of a formula analogous to the one used for air.

Table showing the observed mean velocities in CO_2.

Internal diameter of the tube.	c_{i}. $n=256$.	e_{i}. $n=320$.	g_{i}. $n=384$.	c_{ii}. $n=512$.	c_{iii}. $n=1023{\cdot}25$.
millim.	metre.	metre.	metre.	metre.	metre.
28	255·38	255·73	255·86	256·05	256·37
17·1	254·53	254·96	255·24	255·36	255·78
9·35	252·58	253·03	253·41	253·69	254·49

By combining the results in the vertical columns in pairs as explained on page 254 for air, we find the following values for the velocity of sound in carbonic acid in unlimited space.

Tuning-fork.	Tubes I. and II.	Tubes I. and III.	Tubes II. and III.	Mean.
	millim.	millim.	millim.	millim.
c_{i}	256·7	256·7	256·7	256·7
e_{i}	256·8	257·1	257·2	257·03
g_{i}	256·8	257·2	257·5	257·17
c_{ii}	257·0	257·2	257·2	257·13
c_{ii}	257·0	257·2	257·2	257·13
			Mean	257·03

For the purpose of comparing my results for air and carbonic acid with those of other observers, I have constructed the following table, in which the values for carbonic acid are referred to those for air as unity.

	Dulong *.	Regnault †.	Wüllner ‡.	Kundt §.	Low.
Air	1	1	1	1	1
CO_2	0·7856	0·8009	0·7812	0·7785	0·77750

To test further the validity of Kirchhoff's formula, I have, as before, calculated the products $(a-v)2r$ and $(a-v)\sqrt{n}$ for $a=257{\cdot}03$ metre. The results, taken vertically in the first table and horizontally in the second, should be constant.

TABLE I.

Values of $(a-v)2r$.

Tube.	c_{I}.	e_{I}.	g_{I}.	c_{II}.	c_{III}.
I.	0·04620	0·03640	0·03273	0·02745	0·01848
II.	4275	3540	3061	2856	2137
III.	4161	3741	3385	3123	2374

TABLE II.

Values of $(a-v)\sqrt{n}$.

Tube.	c_{I}.	e_{I}.	g_{I}.	c_{II}.	c_{III}.
I.	26·40	23·26	22·93	22·17	21·11
II.	39·99	37·03	35·09	37·79	39·98
III.	71·24	71·47	70·94	75·56	81·25

Here, again, the deviations from a constant mean are quite unimportant, and are evidently due to small errors of observation.

* *Ann. de Chim. et de Phys.* tom. x. p. 41.
† Regnault, see footnotes, p. 249.
‡ *Lehrbuch der Physik*, iv. Aufl. 1, p. 804.
§ Kundt, see footnotes, p. 249.

The values of γ, the constant for friction and conduction of heat, are as follows:—

Tube.	c_{i}.	e_{i}.	g_{i}.	c_{ii}.	c_{iii}.
I.	0·005099	0·004490	0·004424	0·004284	0·004078
II.	4718	4367	4137	4457	4706
III.	4592	4615	4575	4873	5240
Mean=0·004577.					

The ratio of the specific heats, calculated in the same way as for air, I found to be 1·2883. Röntgen* gives 1·3052 and Müller † 1·2653.

To recapitulate,—

1. Kirchhoff's formula for the determination of the velocity of sound in tubes holds good for carbonic acid as well as for air if the velocity in the free gas = 257·03 metre‡, and the constant for friction and conduction of heat =0·004577.

(2) The ratio of the specific heats for carbonic acid is 1·2883 ‡.

§ 5. *Hydrogen.*

The remodelling of the apparatus for gases lighter than air caused much trouble. In its final form I simply inverted the whole apparatus as employed for carbonic acid. It was now in fact a siphon; the open end of the tube was below and the stopcock above. The few drops of water that trickled out I led aside with a tail of lamp-wick. The method of filling was the same as for carbonic acid, only here everything was inverted.

On account of the extraordinary tenuity of this gas, the wave-lengths are very great, and much longer tubes than before were necessary. I soon found that the energy of the vibrating mass of the gas was too small to set the membrane of the tympanum properly in vibration, and that exact readings of the maxima were unattainable.

The following readings were taken with tube II. and fork c_{ii} at a temperature of 15° C.:—

* Pogg. *Ann.* cxlviii. p. 612 (1874).
† Wied. *Ann.* xviii. p. 116 (1883).
‡ Corrected on page 264.

$\frac{\lambda}{2}$ millim.	v_0 metre.
1185	1260·4
1120	1172·0
1175	1250·6
1160	1234·2
1150	1223·3
1145	1217·9
1220	1297·5
Mean	1236·5

Regnault gives 1200·77 and 1166·67, but these numbers are probably too small.

According to a few experiments which I made later with coal-gas, the form of the apparatus that was used for carbonic acid seems also suitable for gases lighter than air. In this case, however, the observer must operate as rapidly as possible.

§ 6. *Mixtures.*

I also applied the method with good success to mixtures of air and vapours. The form of the apparatus was the same as for air, only that in this case the bottle was filled with the evaporating liquid instead of with water.

I raised the liquid in the tube as far as the side-piece A, then let it slowly sink and remain standing for a time, which varied from a few seconds to two hours. The temperature for all the experiments was constant (17° C.). With tube II. and fork c_{III} I took six sets of readings given below. At the beginning of each set, one or two half wave-lengths come where only very little vapour was present. Then with increasing saturation the half wave-lengths gradually decrease, until in the immediate neighbourhood of the liquid they become constant.

Experiments with Ethyl-ether Vapour.

	1.	2.	3.	4.	5.	6.
		Surface of ether below.				
$\frac{\lambda}{2}$ millim. =	160	153	—	156	153	159
	144	154	145	151	153	156
	130	143	130	132	132	132
	109	110	108	112	110	114
	108	110	114	112	112	112
	112	110	110	113	113	110
	109	109	109	110	110	110
		Surface of ether above.				

The mean of the last six half wave-lengths in the immediate neighbourhood of the surface of the liquid is 109·5 millim.

The velocity of sound in air v is calculated by the formula

$$v^2 = \frac{B \cdot Q \cdot g}{\frac{\sigma}{k}},$$

where B denotes the barometric height, Q the specific gravity of mercury, g the accelerating force of gravity, σ the density of air referred to water, and k the ratio of the specific heats. The analogous formula, when vapour is present, is

$$v^2 = \frac{B \cdot Q \cdot g}{\frac{\sigma}{k} + \frac{\sigma_1}{k_1}}$$

$$= \frac{B \cdot Q \cdot g\,(1+\alpha t)}{\frac{\sigma_0}{k} \cdot \frac{p-S}{p_0} + \frac{\sigma_0}{k_1} \cdot \frac{uS}{p_0}},$$

where u = density of the vapour compared with air, and α = thermal coefficient of expansion of gases.

Or

$$v^2 = \frac{p_0 k (1+\alpha t)}{\sigma_0\left(1 - \frac{S}{p} + \frac{k}{k_1} u . \frac{S}{p}\right)}$$

$$= \frac{p_0}{\sigma_0} k\,(1+\alpha t) . \frac{1}{1+\left(\frac{k}{k_1} u - 1\right)\frac{S}{p}}$$

$$= \frac{B_0 \cdot Q_0 \cdot g \cdot k}{\sigma_0} . \frac{1+\alpha t}{1+\left(\frac{k}{k_1} u - 1\right)\frac{S\ \text{millim.}}{B\ \text{millim.}}};$$

or

$$v = v_0 \sqrt{\frac{1+\alpha t}{1+\left(\frac{k}{k_1} u - 1\right)\frac{S}{B}}}.$$

In further explanation of the above I add

	Air.	Ether vapour.
Density	$\sigma = \frac{1}{773}$	$\sigma_1 = u\sigma = 2{\cdot}60\,\sigma$
$\frac{\text{Sp. heat const. pressure}}{\text{Sp. heat const. volume}}$	k	k_1
Partial pressure	$p - S$	S

$$\text{Pressure } p = B \cdot Q \cdot g,$$
$$p_0 = B_0 \cdot Q_0 \cdot g,$$
$$= 0{\cdot}760 \times 13{\cdot}596 \times 9{\cdot}810 \text{ metre.}$$

If k_1 be the unknown, then

$$k_1 = \frac{ku}{\left\{ \frac{v_0^2}{v^2}(1+\alpha t) - 1 \right\} \frac{B}{S} + 1}$$

By substituting

$k = 1{\cdot}3968$, $\alpha = 0{\cdot}003665$,
$u = 2{\cdot}600$, $t = 17°$ C.,
$v_0 = 330{\cdot}88$ metre, $B = 0{\cdot}750$ metre,
$v = 2 \times 1023{\cdot}25 \times 0{\cdot}1095$ metre, $S = 0{\cdot}38528$ metre,

we get as ratio of the specific heats of ethyl-ether

$$k = 1{\cdot}0202.$$

Jaeger* found 1·097 (at 20°) and Müller† 1·0288 (between 45°·4 and 22°·5).

A phenomenon, similar to the one observed in a mixture of air and vapour, I found also in a mixture of air and carbonic acid.

In the manner already described I filled the tube half full with carbonic acid, then turned the gas off and let the water sink. The upper portion of the tube was thus filled with air and the lower with carbonic acid. With fork c_{III} and tube II. I found the following readings:—

Surface of the water below.

165	165	air.
165	165	
150	140	
135	135	
130	130	carbonic acid.
130	130	

Surface of the water above.

In both columns the two first half wave-lengths agree with those previously found for pure air, the two last with those for pure carbonic acid. This is a simple method of ascertaining with fair exactness the relative velocities of sound in air and carbonic acid or other suitable gases.

* Wied. *Ann.* xxxvi. p. 209 (1889).
† Wied. *Ann.* xviii. p. 116 (1883).

§ 7.

After my experiments were finished I had some doubt as to whether the vibration-frequency of $c_{\prime}$ was exactly 256. I decided the point by means of an electric registration method. A small, thin, pointed piece of platinum-plate was attached to the end of one of the prongs of the fork, and the vibration-curves were traced on a metal cylinder coated with blackened paper. A weak induction-current, with mercury contact with the seconds-pendulum of a clock of known daily error, was generated and conducted by the platinum point to the blackened paper. The number of waves between the marks of the sparks, taken two and two, gave the double vibration-frequency of the fork. As a result I found that $c_{\prime}$ made 256·23 vibrations per second. This necessitated small corrections in my results, which are shown below.

Velocity of Sound.

	Uncorrected.	Corrected.	Regnault *.
Air..................	330·582 m.	330·88 m.	330·7 m.
Carbonic Acid ...	257·03 m.	257·26 m.	259·57 m.
Hydrogen	1236 5 m.	1237·6 m.	1200·77 & 1166·67 m.
Ether Vapour ...	175·77 m.	175·93 m.	178 8 m.†

Corresponding corrected Ratios of the Specific Heats.

	Low.	Röntgen ‡.	P. A. Müller §.
Air..................	1·3968	1·4053	1 4062
Carbonic Acid ...	1·2914	1·3052	1·2653
Hydrogen	1·3604	1·3852	......
Ether Vapour ...	1·0244	......	1·0288 (at 22°) 1·094 †

* *Mém. de l'Inst.* xxxvii. p. 133; *Compt. Rend.* lxvi. p. 219 (1868).
† Jaeger, Wied. *Ann.* xxxvi. p. 209 (1889).
‡ Pogg. *Ann.* cxlviii. p. 606 (1873).
§ Wied. *Ann.* xviii. p. 116 (1880).

The experiments with hydrogen can only be regarded as approximately correct.

My results for the velocity of sound in air and carbonic acid in glass tubes of different diameters are in full agreement with Kirchhoff's theoretical formula founded upon the consideration of the friction and the conduction of heat of gases.

The velocity of sound in free space for air and carbonic acid is, according to my results, invariable for tones of different pitch and intensity.

It is my pleasant duty to express here to Professor Quincke my heartiest thanks for his kind support and instructive counsel during the prosecution of the above inquiry.

The Physical Institute, Heidelberg,
January 1894.

XXIX. *The Hatchet Planimeter.* *By* F. W. Hill*.

THE hatchet planimeter consists essentially of a tracing-point and a convex chisel-edge rigidly connected, the point and the edge being in the same plane. When the point is moved along any line, the edge describes a curve of pursuit.

The object of this paper is to investigate how the instrument may be used to determine areas.

Let the tracer start from a point O inside the area, move along any line to the perimeter, then round the perimeter and back along the same line to O; the solution of the problem consists in finding an expression for the angle AOB between the initial and final positions OA, OB of the line joining the tracing-point and chisel-edge.

All attempts to express the area of the curve in terms of this angle proved futile except in a few special cases, such as the circle and square; but the magnitude of the angle may be found in the form of an infinite series, the most important term of which is a multiple of the area. The complexity of the result would seem to show that no simple geometrical explanation is possible.

Let the tracer move a distance r along a straight line (fig. 1); then, if χ, χ' be the initial and final inclinations of the rod to the line, c the length of the rod, it is easily proved that

$$\tan\frac{\chi'}{2}=e^{-\frac{r}{c}}\tan\frac{\chi}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

* Communicated by the Physical Society: read June 22, 1894.